Public Banks Improve Private Banks Performance: Evidence from a Dynamic Structural Model *

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Abstract

This paper shows that profits of private banks are positively affected by the number of public branches operating in Brazilian isolated markets. The spill-over generated by public banks is quantified based on a dynamic oligopoly model. A counterfactual in which public banks are privatized is examined. It shows that the number of active branches operating in the long-run in a small market drops significantly. Our paper also improves on the existing literature by developing a new estimation method for dynamic games. We show that the OLS estimator yields consistent estimates of the structural parameters, which reduces significantly the computational burden.

1 Introduction

The discussion about the existence of public, state owned banks has been prominent in the banking literature since the 1960's - see Barth, Caprio and Levine (2001) and La Porta, López-de-Silanes and Shleifer (2002) and Levy Yeyati, Micco and Panizza (2007).

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In favour of public banks the following has been argued: (i) Public banks finance unprofitable but socially valuable investment projects; (ii) they foster financial development and (iii) they provide financial access to populations living in areas that are unattractive for private institutions. Critics of public banks argue that (i) they are used as political instruments, providing employment, credit, subsidies or other benefits in return for political assistance, and (ii) they crowd-out more efficient, more competitive private banks, slowing down the development of the financial system.

This paper examines whether public banks complement or crowd-out private banks. A dynamic game between the major Brazilian public and private banks is estimated. A counterfactual experiment is used to analyze how the privatization of public banks affects the supply of banking services in small isolated markets.

Two main conclusions emerge. First, public banks generate positive profit spill-overs for private banks; second, private banks crowd-out private competitors. Our estimates show that the entry of a public bank in a given market increases the return of a private incumbent by 1.2 percent and the entry of a private bank reduces the return of a private incumbent by 0.05 to 1 percent.

The counterfactual in which public banks are sold to private players shows that the total number of active branches operating in the long-run in a typical small market drops from 3 to 0.5 on average. To guarantee that, after privatization, all small municipalities would have at least one active branch the government should give a subsidy of approximately 8% on the operational costs of private branches. We can infer that the present cost of this policy would be of approximately US\$175,000¹ per market. This value is relatively small compared to the market value of Brazilian public banks. Bank of Brazil, the largest public bank in Brazil, had its market value estimated in approximately US\$42 billions in 2012. This means that the resources raised with the privatization of Bank of Brazil would be sufficient to cover the subsidies for 240 thousand branches in the country or, approximately, 42 branches per Brazilian municipality.

These findings have important policy implications in developing countries. In these countries a large fraction of the population has no access to the banking market. Yet the access to financial services generates positive effects in terms of poverty reduction and economic growth in disadvantaged areas (Burgess and Pande (2005) and Pascali (2012)).

Our estimates do not allow us to disentangle the details of the spill-over channels. Broadly speaking, our findings are consistent with public banks (i) having monopoly over a number

¹Approximately R\$350,000.

of important Federal funds and (ii) being driven by social, as opposed to strategic or market reasons. The first element guarantees a large volume of credit for small markets - see Feler (2012). The second induces product differentiation between public and private banks: Public and private banks target different clients - see Coelho, Melo and Rezende (2012). In this case, the amount of cheap credit and public transfers poured by the public banks in small isolated municipalities shifts the demand for banking services, making these markets more attractive for private players. This effect induces the entry of private players.

There is little prior empirical evidence of the effects of public banks on the economy. The evidence is mixed. La Porta, López-de-Silanes and Shleifer (2002) study a cross section of countries and show that the presence of public banks in the market is associated with poorly developed financial markets. They conclude that the higher the public ownership in the banking sector, the lower is the average growth of the ratio of private credit to GDP. Similar findings were obtained in Barth, Caprio and Levine (2001) who find that greater state ownership of banks with more poorly developed banks, nonbanks, and securities markets.

Levy Yeyati, Micco and Panizza (2007) extend the dataset used in La Porta, López-de-Silanes and Shleifer (2002) by including more controls and a longer period of time. They find that no robust conclusion can be drawn. The findings depend strongly on the definition of financial development, the estimator and the sample definition. They conclude that there is "(...) no significant correlation between state-ownership of banks and credit to the private sector"². Detragiache, Tessel and Gupta (2008) confirm the findings in Levy Yeyati, Micco and Panizza (2007).

The effects of public banks on development are analyzed in Cole (2007) and Feler (2012). Cole (2007) finds that the nationalization of public banks in India increased the amount of credit but had no effect on real outcomes. Feler (2012) analyses the privatization of state banks in Brazil. His findings are close to the findings in Cole (2007): Privatization led to a significant reduction of credit supply in local markets but it did not affect local GDP.

Coelho, Mello and Rezende (2012) analyze the effects of public banks on the performance of private banks. They extend the traditional Bresnahan and Reiss (1991) framework and estimate an empirical model using a cross-section of entry and exit movements in Brazilian municipalities. This approach is related to ours, but in that paper the analysis is static and relies purely on cross-sectional variation. Based on the negative and significant but small coefficient related to the number of public banks in the profit function of private banks, they conclude that public banks do not affect competition in the market.

²Levy Yeyati, Micco and Panizza (2007).

This paper builds a dynamic entry game in which the major Brazilian public and private banks are the players. The dynamic structure of the model is strongly supported by our data and can be intuitively rationalized by the existence of substantial entry costs in the market. At each period these players have information about the state variables and decide simultaneously to be active or not active in a given market by maximizing an inter-temporal profit function. Entrants pay a fixed cost. We assume that the profit function of the major private players is asymmetrically affected by public and private competitors. This allows us to understand how public banks influence the performance of private players.

We use data from 1002 isolated markets in Brazil during 1988-2010 to estimate the decision rules for public and private banks. We infer the model primitives that rationalize these decision rules in a dynamic oligopoly game. Importantly, we evaluate the market equilibrium under different counterfactual scenarios. We report consistent *ex ante* estimates of the effects of changes in the banking market structure on market outcomes. Our model is valuable to predict policy changes. By relying on micro data from a single market, we are able to reduce the market heterogeneity present in cross country regressions, which, as reported by Levy Yeyati, Micco and Panizza (2007), causes important bias in the conclusions obtained by the existing literature.

Methodologically our paper is related to the empirical industrial organization literature that studies the estimation of dynamic games - see Aguirregabiria and Nevo (2010), Bajari, Hong and Nekipelov (2010) and Pesendorfer (2010) for a rich discussion on the topic. Applications that are similar to ours are also found in Pesendorfer and Schmidt-Dengler (2003) for small businesses in Austria, Dunne, Klimek, Roberts and Xu (2009) for dentists and chiropractors in the US, Gowrisankaran, Lucarelli, Schmidt-Dengler and Town (2010) for hospitals in the US, Collard-Wexler (2010) for the concrete industry in the US, Ryan (2012) for the cement industry in the US and Kalouptsidi (2011) for the shipping industry. Other applications include Maican and Orth (2012), Minamihashi (2012), Lin (2011), Fan and Xiao (2012), Nishiwaki (2010), Arcidiacono, Bayer, Blevins, and Ellickson (2012), Jeziorski (2012), Snider (2009), Suzuki (2012), Sweeting (2011) and Beresteanu, Ellickson and Misra (2010).

Our estimation procedure improves Pesendorfer and Schmidt-Dengler (2008). Our methodological innovation is to show that the OLS estimator gives us consistent estimates of the structural parameters. This approach allows us to avoid the use of numerical methods in the estimation of the structural parameters. Doing so we reduce significantly the computational burden. This paper is organized as follows. The next section describes our dataset and the Brazilian banking market. Section 3 shows reduced form evidence of competition between public/private players. Sections 4 and 5 describe the theoretical model, the empirical model and our main results. Section 6 discusses the fitting of the empirical model and our counterfactual analysis. The last section concludes the paper.

2 Data and Institutional Background

The data come from the Brazilian Central Bank and from the Brazilian Ministry of Labour. The Brazilian Central Bank database follows the activities of all Brazilian banks since 1900. These data contain the opening and closing dates³ and the name of the chain that operates each branch for all branches opened since 1900 in all Brazilian municipalities. A measure of market size is constructed by using data from the Brazilian Ministry of Labour containing the total payroll in the formal sector⁴ for all Brazilian cities since 1985. The payroll data is deflated using the official inflation index, IPCA-IBGE. All the values are in R\$ of 2011. The information about banking and economic activity is annual.

Following Bresnahan and Reiss (1991) our analysis examines small isolated markets. We select municipalities⁵ that are at least 20 km away from the nearest municipality. State capitals and metropolitan areas are excluded. We also excluded municipalities that had more than 10 bank branches since 1900. This selection leaves us with 1002 isolated small markets, corresponding, roughly, to 20% of all Brazilian municipalities. The idea of isolated markets enable us to obtain a clear measure of the potential demand for each branch.

The market size data starts in 1985. We exclude 1986 and 1987 from our sample because a major macroeconomic shock caused by two heterodox stabilization plans⁶ disorganized severely the Brazilian economy in those years. Our final sample consists of observations for 1002 isolated municipalities in the period 1988-2010. Because in most of our municipalities the chains have at most one branch we focus on entry and exit patterns.⁷

Sample statistics. The next table illustrates the basic statistics of our sample.

 $^{^3\}mathrm{For}$ the branches that were closed.

⁴Number and wage of employees in the formal sector of the economy.

⁵From now on we use municipality/market interchangeably.

⁶Cruzado Plan in 1986 and Bresser Plan in 1987.

⁷In the municipalities that had more than one branch operated by the same chain, which correspond to less than 4% of the total number of municipalities and around 0.2% of our sample, we aggregated the branch level information for each player that had more than one branch in the same market. The exclusion of these municipalities does not change our results. Therefore we kept this information in the dataset.

	Average	Std
Active Branches (**)	1560.2	142.4
Entry $(**)$	50.5	49.1
Exit (**)	59.5	55.1
Sample Market Size (*) (**)	10,404	.85
Municipalities	100	2
Sample Municipalities/Total Number of Municipalities	18%	0
Municipalities \times Periods	2204	1

Table 1: Basic Sample Statistics 1988-2010

Note: * R\$ millions of 2011. ** Yearly averages.

Our sample is composed by 1002 isolated markets. This corresponds to approximately 18% of the total number of municipalities in Brazil. The number of branches in this sample is 1560 per year on average. Entry is observed 50 times per year and exit 59 times. The yearly market size measured by the annual payroll of the formal workers of all the municipalities in the sample is of R\$ 10.4 billions of 2011. This value is relatively small because by excluding state capitals and metropolitan regions the richiest cities in the country are left aside.

The Brazilian banking market is basically dominated by four big institutions: Two of them, Bank of Brazil, BB, and Caixa Economica Federal, CEF, are public and controlled by the federal government while the other two, Bradesco and Itau, are privately held. The next figures show the number of branches that are controlled by these institutions and the market share, measured in terms of the number of active branches in the sample, of these four players.



Figure 1: Number of Branches (left) and Market Share (right) - "Big" Four

Note: Number of active branches per year (left) and fraction (right) of these branches over the total number of active branches in our sample.

These four players hold more than 80% of the active branches in our sample. The share of

Bradesco and Itau increased substantially over time. From 2000 to 2010 Bradesco's market share measured in terms of active branches increased from 13% to 20%. Itau's share increased from 1% to 10%. Part of the expansion is explainable by the acquisitions of privatized smaller public institutions. Bank of Brazil, BB, also experienced an increase in the number of branches. This expansion is mainly driven by the social policy of Lula's government (2003-2010), which tried to expand the presence of public banks in small markets.

	ng i agron ana i s		
Number of Private	Number of Public	Mun Payroll	Observations
0	0	0.227	4592
0	1	0.478	6496
0	2	0.680	2511
1	0	0.515	1555
1	1	1.333	1696
1	2	1.677	1107
2	0	1.034	149
2	1	2.419	552
2	2	2.202	380

Table 2: Average Monthly Payroll and Number of Public/Private Players

Note: Average market size is the monthly average payroll of the municipality and is measured in R\$ millions of jan/2011 according to the number of players in the market. Sample period: 1988-2010. Each observation corresponds to a municipality in a given year. We showed in the table only the most frequent market structures. This correposeds to around 90% of the total number of observations.

Table 2 reports (i) the frequency distribution of each market configuration (number of observations corresponding to each market structure) and (ii) the average market size (monthly average payroll of the municipality in R\$ millions of 2011) corresponding to each market structure. These numbers illustrate that:

- 1. Public players are located more frequently in small markets (as measured by the municipality average payroll) than private players; and,
- 2. Public players are frequently the only providers of financial services in these isolated markets (the frequency of public monopolies 6496 observations is the highest in the sample).

The Brazilian government has launched some programs that aim to "popularize" basic financial services in poorer areas, which includes the supply of basic services (current account, for example) and of credit lines to small farmers and firms. This feature may explain the fact that the frequency of markets where the public player is a monopolist (the only provider of financial services) is quite high in our sample. In addition, the empirical evidence indicates that public banks are much less productive than their private counterparts (Nakane and Weintraub (2005)). This means that the presence of public banks in smaller markets is not explained by cost advantages of public players.

■ Institutional background. The Brazilian banking market is large. In 2012 Itau was considered the 8th largest bank in the world in terms of market value (with a market value of US\$88 billions); Bradesco was the 17th largest (market value of US\$64 billions) and Bank of Brazil was the 31st largest (market value of US\$42 billions)⁸.

As pointed out in Coelho, Melo and Rezende (2012) there are important differences in the objectives of public and private banks. Private banks are essentially profit oriented. By legal mandate public banks focus their operations on market segments that are not profitable for private banks. This suggests the existence of product differentiation in the market. In what follows we describe the "social" role of public banks in Brazil⁹.

Bank of Brazil (BB) has expanded enormously its operations in smaller and poorer areas of the country based on central government policies aiming to "popularize" banking services among poor workers and small businesses. BB plays an important role as the provider of government funds to the Brazilian agriculture¹⁰. Also, to expand its capillarity in isolated areas, BB created a DSR (Regional Development Program). The DSR provides a set of tools for small entrepreuners, including a business plan, technical support and credit¹¹.

Caixa Economica Federal (CEF) has monopoly over a number of different government funds and services, such as the FGTS, Bolsa Família, PIS¹² and the Federal Lottery¹³. FGTS is a Brazilian fund created in 1966 to provide assistance to unemployed people¹⁴. These

⁸http://www.relbanks.com/worlds-top-banks/market-cap. Access: November 12, 2012.

⁹also present a detailed discussion on the role of Brazilian public banks.

¹⁰The total amount of agricultural credit provided by this player in 2010 reached more than US\$ 26 billions. Moreover, BB is the main bank in the Pronaf, a program created to supply credit for small businesses (agriculture, fishing, turism, and handcraft) in rural areas at a very low interest rate. The total credit available for the program increased from US\$ 1 billion in 1999 to US\$ 7 billions in 2010. All banks in Brazil are allowed to take part in the program, however, BB distributes around 65% of the total Pronaf credit.

¹¹In 2007 the program supported 2800 business plans and distributed US\$ 1.7 billions in credit.

¹²PIS is a tax to cover unemployment benefits. Their assets were around US\$14 billions in December 2010. ¹³The Federal Lottery provided a gross revenue of US\$5.2 billions in 2010. It is used to fund sports.

¹⁴The main source of funding is the monthly compulsory deposit that every private employer must do in the name of each employee. These values constitutes a fund and the worker have access to the money deposited in his/her name only in some special conditions: Unemployment, chronic disease, and for buying a house (if the worker does not own another house). CEF is responsible for the whole operation of the fund - from the tax collection to the payments for the benefited workers. Since the FGTS universe includes all formal workes (except public servants) the total size of the fund is considerably high - around US\$90 billions

resources are allocated in two main areas: Housing and sanitation. The government gives the investments guidelines in order to finance strategic areas with lack of credit. CEF is also responsible for the distribution of the benefits from Bolsa Família¹⁵, a program that gives to poor families a monthly income. It was created to reduce the poverty in the most backward areas of the country.

Summarizing, the descriptive analysis suggests that the major Brazilian public banks have been used by the federal government to improve the financial access of isolated markets. The financial access includes new branches in smaller and poorer markets and the injection of cheap credit in these areas. These operations are not profitable for private banks. This suggests the existence of product differentiation in the market.

3 Reduced Form Analysis

We estimate a series of reduced form logit models to explain entry/exit movements of the biggest public and private players using the sample of isolated municipalities. We focus on the behavior of the 4 largest players: Bank of Brazil (BB), Caixa Economica Federal (CEF), Bradesco and Itau.

Two pooled logit models are estimated: One for the public players, BB and CEF; the other for private players, Itau and Bradesco. Our logit specification is:

$$P(a_{imt} = 1 | a_{imt-1}, n_{mt-1}^{pub}, n_{mt-1}^{pri}, \mathbf{x_{mt}}; \rho, \mu) = \Lambda(\rho_0 + \rho_1 a_{imt-1} + \rho_2 n_{mt-1}^{pub} + \rho_3 n_{mt-1}^{pri} + \rho_4 \mathbf{x_{mt}} + \mu_t + \mu_m + \mu_{mt} + \mu_i)$$
(1)

The dependent variable, a_{imt} , is the action of player *i* in municipality *m*, period *t*. It assumes 1 if player *i* was active in that municipality/period and zero otherwise. a_{imt-1} indicates the action of the same player in that municipality in the prior period. n_{mt-1}^{pub} is the number of **public** competitors in the previous period in market m^{16} ; n_{mt-1}^{pri} is the number of

¹⁶Mathematically, $n_{mt-1}^{pub} = \sum_{j \in i_{pub}, j \neq i} a_{jmt}$, where i_{pub} is the set of public players.

in December 2008.

 $^{^{15}}$ In 2006 the program served around 11 million families or approximately 44 thousand individuals. The public expenditure with the program is around 0.5% of the Brazilian GDP and is growing steadily since its creation.

private competitors in the previous period in market m^{17} ; \mathbf{x}_{mt} is a vector of municipality characteristics; μ_t are time effects; μ_m are market specific effects; μ_{mt} captures market/time specific effects and μ_i are player specific effects. $\Lambda(\cdot)$ is the logistic distribution. The greek letters denote parameters to be estimated. The data include all muncipalities where player i was active for at least one period¹⁸.

The estimation of dynamic binary response models with market fixed effects produces biased coefficients - see Carro (2007), Wooldridge (2010). To avoid this bias in our analysis, instead of including market dummies, 27 state dummies (one for each Brazilian state) were included in the model. States dummies are used to capture time invariant heterogeneity across municipalities of different states. Time effects are captured by year dummies. Time varying market effects are captured by the interaction of state dummies and a trend variable.

The vector \mathbf{x}_{mt} includes municipality payroll, transfers of the Federal and State governments to the municipality, municipal government expenditure and agricutural production of the municipality. Municipality payroll is a measure of market size. The inclusion of transfers and municipal expenditure controls for the fact that entry of public banks can be correlated with an increase of Federal/State investment in the municipality, which also can affect entry of private banks. Agricultural production is included because a large fraction of the income in our isolated municipalities comes from agricultural activities. This variable is a different indicator of market size.

3.1 Private players

Table 3 reports the estimates of equation (1) for the private players, Bradesco and Itau. Only the marginal effects of n_{mt-1}^{pub} and n_{mt-1}^{pri} evaluated at the sample means are reported. The model fit is good, with Pseudo-R2 of 87%-91%. Strikingly, the number of public banks increases the entry probabilities of the private players by 10%-14%. The effects are very significant and robust across specifications. The inclusion of state dummies and the interaction between state dummies and the time trend increases this effect.

Interestingly, the number of private competitors reduces the entry probabilities of private

¹⁷Mathematically, $n_{mt-1}^{pri} = \sum_{j \in i_{pri}, j \neq i} a_{jmt}$, where i_{pri} is the set of private players.

¹⁸Our estimation approach is based on the potential markets for each player. The potential market is defined based on the super efficient estimator described in Pesendorfer and Schmidt-Dengler (2003). We defined that market m is a potential market for player i if $\max_{t} \{a_{imt}, t = 1900, 1901, ..., 2010\} = 1$, or, in other words, market m is a potential market for player i if she entered in that market at least for one period since 1900.

players. This effect is around -6.4% in the specification with the full set of controls. It is statistically significant at 5%.

Table 3: Marginal Effects of n_{mt-1}^{pub} and n_{mt-1}^{pri} on the Entry Probabilities of Private Players, Bradesco and Itau

	(I)	(II)	(III)	(IV)
N ^o Public	0.10544^{***}	0.12839^{***}	0.13829^{***}	0.13237***
	[0.01]	[0.02]	[0.02]	[0.02]
$N^{\underline{o}}$ Private	-0.03532	-0.01930	-0.06206**	-0.06409**
	[0.02]	[0.03]	[0.03]	[0.03]
Player Dummy	Yes	Yes	Yes	Yes
Time Dummies	Yes	Yes	Yes	Yes
State Dummies	No	Yes	Yes	Yes
Trend*State Dummies	No	No	Yes	Yes
Transfers, Expenditure, Agric. Prod.	No	No	No	Yes
Observations	15,919	15,229	15,229	15,217
Pseudo R2	0.87	0.87	0.91	0.91

Note: (***) Significant at 1%; (**) significant at 5%. Marginal effects calculated at the sample means. Clustered standard errors in brackets. All the models have lagged activity, number of public and private competitors and municipality payroll. Transfers correspond to the total transfers of Federal and State governments to the municipality. Expenditure corresponds to municipal government expenditure. Agricultural Production is the total agricultural production of each municipality.

As robustness check, the model was estimated in a subsample of municipalities that had (i) at least one public player and (ii) at least one and at most three public players in any time period. Tables 9 and 10 in the appendix report the results for each subsample.

The pattern of results remains unchanged when compared to the estimates in Table 3. This strategy is used to control for unobservable characteristics of markets with and without public players. In the subsample with at least one and at most three public players the market heterogeneity is reduced. Markets with roughly the same number of public players have similar observable characteristics.

3.2 Public players

Table 4 reports the estimates of equation (1) for the public players, Bank of Brasil and Caixa Economica Federal. The model fit is good, with Pseudo-R2 statistics around 85%. Strikingly, entry probabilities of public players are barely affected by the number of public and the number of private competitors in each market. Although significant in some specifications the marginal effects of the number of public and the number of private competitors are very small when contrasted with the estimates in Table 3. In all the specifications the marginal

effects of n_{mt-1}^{pub} and n_{mt-1}^{pri} are small in magnitude, being below 1% and 0.5% respectively. In specification (IV) in Table 3 these effects were respectively 13% and -6.4%. These estimates imply that the marginal effects of n_{mt-1}^{pub} and n_{mt-1}^{pri} are around 13 times larger for private banks than for public banks. This pattern is robust to the inclusion of state dummies, the interaction of state dummies and the time trend variable, public transfers, municipal expenditure and agricultural production.

	(I)	(II)	(III)	(IV)
N^{Ω} Public	0.00498***	0.00871***	0.00934***	0.00936***
	[0.00]	[0.00]	[0.00]	[0.00]
$N^{\underline{O}}$ Private	0.00016	0.00105	0.00455^{**}	0.00455^{**}
	[0.00]	[0.00]	[0.00]	[0.00]
Player Dummy	Yes	Yes	Yes	Yes
Time Dummies	Yes	Yes	Yes	Yes
State Dummies	No	Yes	Yes	Yes
Trend*State Dummies	No	No	Yes	Yes
Transfers, Expenditure, Agric. Prod.	No	No	No	Yes
Observations	20,357	$20,\!357$	20,357	20,350
Pseudo R2	0.83	0.84	0.87	0.87

Table 4: Marginal Effects of n_{mt-1}^{pub} and n_{mt-1}^{pri} on the Entry Probabilities of Public Players, BB and CEF

Note: (***) Significant at 1%; (**) significant at 5%; (*) significant at 10%. Marginal effects calculated at the sample means. Clustered standard errors by municipality in brackets. All the models have lagged activity, number of public and private competitors and municipality payroll. Transfers correspond to the total transfers of Federal and State governments to the municipality. Expenditure corresponds to municipal government expenditure. Agricultural Production is the total agricultural production of each municipality.

4 Theoretical Model

This section sets up and solves a dynamic entry game between the major Brazilian banks. Motivated by the data, the game considers entry and exit decisions. In the data a chain has typically at most one branch in each municipality¹⁹. We focus on the behavior of two public banks, Bank of Brazil and Caixa Economica Federal, and two private banks, Bradesco and Itau. In 2010, these players had more than 80% of the total number of active branches in our sample.

The model captures the features documented by the reduced forms. Dynamics can be rationalized by high entry $costs^{20}$. Importantly, the model allows for different behavior of

 $^{^{19}\}mathrm{As}$ described above only in 4% of these municipalities one chain had more than 1 branch during the same period.

 $^{^{20}}$ Market analysts point out that the returns of branches in small markets is quite low. Lower returns in

public and private players.

We estimate the primitives that rationalize the behavior of private banks using a dynamic oligopoly game. We do not structurally model the behavior of public banks. Entry decisions of public banks are assumed to do not depend on the number of public and the number of private competitors in the market. There are two explanations behind this assumption. First, the reduced form analysis suggests that entry probabilities of public banks are barely affected by the number of public and the number of private competitors in the market. Second, the literature recognizes that public banks are not necessarily profit maximizers. The behavior of public banks can depend on political and social reasons - see Levy Yeyati et al (2007), La Porta et al (2002) and Barth et al (2001).

At each period private players have information about the state variables and decide simultaneously to be active or not active in a given market by maximizing an inter-temporal profit function. Private players know that the entry of public banks do not depend on the actions of public and private competitors. Private entrants pay a fixed cost. The profit function of the major private players is assumed to be asymmetrically affected by public and private competitors. This allows us to understand how public banks influence the performance of private players.

Closed related models were applied in Pesendorfer and Schmidt-Dengler (2003), Dunne, Klimek, Roberts and Xu (2009), Gorisankaran, Lucarelli, Schmidt-Dengler and Town (2010), Collard-Wexler (2010), Ryan (2012) and Kalouptsidi (2011), among others. Aguirregabiria and Nevo (2010), Bajari, Hong and Nekipelov (2010) and Pesendorfer (2010) present a rich discussion on the estimation of dynamic games.

4.1 Assumptions

■ Players. There are two private players, Bradesco and Itau. The set of private players is $i_{pri} \in \{Bradesco, Itau\}$. There are two public players, Bank of Brazil, BB, and Caixa Economica Federal, CEF. The set of public players is $i_{pub} \in \{BB, CEF\}^{21}$.

Time and markets. Time is discrete, $t = 1, 2, ..., \infty$ and there are $m \in \mathbf{M} = \{1, 2, 3, ..., \overline{M}\}$

these markets are explained by high fixed and operational costs and by reduced revenues - see Gonçalves and Sawaya (2007), Gouvea (2007) and Andrade (2007). This explains why the number of bank branches is small in the most backward areas of the country.

²¹We also estimated a version of the model including a fringe of public/private players. The inclusion of these players does not change our results but increases substantially the state space of our model. This imposes computational difficulties to solve the model and to make conterfactual analysis. By this reason we do not include these players in the model.

markets.

• Actions. A player's action in market m, period t is denoted by $a_{im}^t \in \{0, 1\}$, where 0 means that player is inactive; 1 means that player is active. The $1 \times N$ vector $\mathbf{a}_{\mathbf{m}}^{\mathbf{t}}$ denotes the action profile in market m, period t. We sometimes use $\mathbf{a}_{-\mathbf{im}}^{\mathbf{t}}$ to denote the actions of all players but player i.

State space. The state space is discrete and finite. The state variables consist of a vector $\mathbf{x}_{\mathbf{m}}^{\mathbf{t}}$, of exogenous variables, and of the previous period actions. We call the vector of states $\mathbf{s}_{\mathbf{m}}^{\mathbf{t}} = (\mathbf{x}_{\mathbf{m}}^{\mathbf{t}}, \mathbf{a}_{\mathbf{m}}^{\mathbf{t}-1})$. The state variables are publicly known to the players and to the econometrician.

Transitions. The vector $\mathbf{s}_{\mathbf{m}}^{\mathbf{t}}$ evolves according to the transition matrix $p_m^s(\mathbf{s}_{\mathbf{m}}^{\mathbf{t+1}}|\mathbf{s}_{\mathbf{m}}^{\mathbf{t}}, \mathbf{a}_{\mathbf{m}}^{\mathbf{t}})$, described by next period distribution of possible values for the vector $\mathbf{s}_{\mathbf{m}}^{\mathbf{t}}$ conditional on each possible current state and actions in municipality m.

Unobservables. In each period players draw a profitability shock ε_{im}^t . The shock is privately observed while the distribution is publicly known.

Payoffs of private players. *Private* player's period payoff is:

$$\Pi(\mathbf{a}_{\mathbf{m}}^{\mathbf{t}}, \mathbf{s}_{\mathbf{m}}^{\mathbf{t}}; \boldsymbol{\Theta}_{\mathbf{i}\mathbf{m}}) = \begin{cases} \pi_{im}(\mathbf{a}_{\mathbf{m}}^{\mathbf{t}}, \mathbf{x}_{\mathbf{m}}^{\mathbf{t}}) \\ +\mathbf{1}(a_{im}^{t} = 1) \cdot \varepsilon_{im}^{t} \\ +\mathbf{1}(a_{im}^{t} = 1) \cdot \mathbf{1}(a_{im}^{t-1} = 0) \cdot F_{i} \end{cases}$$
(2)

Here $\pi_{im}(\mathbf{a_m^t}, \mathbf{x_m^t})$ denotes player *i's* deterministic profits in market *m*, *F_i* are fixed costs and ε_{im}^t is a profitability shock. Θ denotes the parameters in the model including *F_i*.

This specification captures the main aspects of our idea. In this formulation, an incumbent that decides to stay in the market have profits given by $\pi_{im}(a_{im}^t = 1, \mathbf{a_{-im}^t}, \mathbf{x_m^t}) + \varepsilon_{im}^t$. An entrant have the same profit as the incumbent plus a negative entry cost F_i . That is, any player that was outside the market and decides to entry pays a fixed cost F^{22} .

The term $\pi_{im}(\mathbf{a_m^t}, \mathbf{x_m^t})$ is a linear function of exogenous states and actions²³:

 $^{^{22} \}rm We$ assume that players leaving the market get a scrap value equal to zero. This hypothesis was also used in Collard-Wexler (2010).

²³Similar structures were used in Pesendorfer and Schmidt-Dengler (2003, 2008), Ryan (2012) and Collard-Wexler (2009).

$$\pi_{im}(\mathbf{a_m^t}, \mathbf{x_m^t}) = \left\{ \pi_{0i} + \pi_{1i}^{pub} \left(\sum_{j \in i_{pub}} a_{jm}^t \right) + \pi_{1i}^{pri} \left(\sum_{j \neq i, j \in i_{pri}} a_{jm}^t \right) + \pi_{2i} x_m^t \right\} \cdot \mathbf{1}(a_{im}^t = 1)$$
(3)

Here $\pi_{ji}^k \in \mathbb{R}^k$ are parameters and x_m^t is a demand shifter. This specification allows for different "competition" effects of public and private players.

The profitability shock ε_{im}^t is assumed to have three components:

$$\varepsilon_{im}^t = \mu_{im} + \eta_{it} + \xi_{im}^t$$

Here μ_{im} is a term that varies only across markets and players but not over time, η_{it} is a time varying player specific term and $\xi_{im}^t \sim EV(0,1)$ is an idiosyncratic shock iid across individuals, time and markets. This is the only source of asymmetric information in the model. The first and the second elements of ε_{im}^t are known to the players and capture respectively (i) the correlation of the profitability shocks in the same market across time and (ii) correlation of the profitability shock across time in different markets. Both effects are empirically justified by the significance of state and year dummies in the reduced forms analyzed above.

The time varying shock is included to capture the fact that the decision structure of the chains can be centralized: First, the "general" conditions of the economy are observed; second the decision in which municipality(ies) to enter/exit is taken. The model captures the feature that a better (worse) macroeconomic landscape can increase (decrease) the probability of being active in all available markets.

We impose a structure in the time effect. We further assumed that $\eta_{it} = \eta_i \bar{x}_t$, where $\bar{x}_t = \sum_m x_{mt}$, is the total payroll of the municipalities in our sample in a given year. The process for the shock is:

$$\varepsilon_{im}^t = \mu_{im} + \eta_i \bar{x}_t + \xi_{im}^t$$

The parameters of interest are $\Theta_{im} = \left\{ F_i, \pi_{0i}, \pi_{1i}^{pub}, \pi_{1i}^{pri}, \pi_{2i}, \mu_{im}, \eta_i \right\}.$

We do not structurally model the behavior of public banks. Entry decisions of public banks are assumed to do not depend on the number of public and the number of private competitors in the market. We do not specify the payoff structure of public players.

- **Sequence of decisions.** The sequence of events of the game is the following:
 - 1. States are observed by all the players.
 - 2. Each player draws a private profitability shock ε_{im}^t .
 - 3. Actions are simultaneously chosen. Private players maximize their payoffs given beliefs on competitor's actions. The total payoff of a private player is given by the discounted sum of player's period payoffs. The discount rate is given by $\beta < 1$ and is the same for all players.
 - 4. After actions are chosen the law of motion for $\mathbf{s_m^t}$ determines the distribution of states in the next period; the problem restarts.

Next the equilibrium for this game is characterized.

4.2 Equilibrium characterization

We restrict our attention to pure *Markovian strategies*. This means that players' actions are fully determined by the current vector of state variables. Intuitively, whenever a player observes the same vector of states it will take the same actions and the history of the game until period t does not influence player's decisions.

■ **Public players.** Public players are assumed to be exogenous. Entry probabilities of public players are known to the private players and do not depend on the actions of other public/private players.

■ **Private players.** Private player *i*'s best response solves the following Bellman equation:

$$\operatorname{Max}_{a_{im}^{t} \in \{0,1\}} \sum_{\mathbf{a}_{-\mathbf{im}}^{t}} \sigma_{im}(\mathbf{a}_{-\mathbf{im}}^{t} | \mathbf{s}_{\mathbf{m}}^{t}) \left\{ \begin{array}{c} \Pi(\mathbf{a}_{\mathbf{m}}^{t}, \mathbf{s}_{\mathbf{m}}^{t}; \boldsymbol{\Theta}_{\mathbf{im}}) + \\ \beta \cdot \sum_{\mathbf{s}_{\mathbf{m}}^{t+1}} p_{m}^{s}(\mathbf{s}_{\mathbf{m}}^{t+1} | \mathbf{s}_{\mathbf{m}}^{t}, \mathbf{a}_{\mathbf{m}}^{t}) \cdot V(\mathbf{s}_{\mathbf{m}}^{t+1}; \sigma_{im}(\cdot), \boldsymbol{\Theta}_{\mathbf{im}}) \end{array} \right\}$$

$$(4)$$

Here $\Pi(\mathbf{a_t}, \mathbf{s_t}; \boldsymbol{\Theta})$ is given by (2) and (3) and $V_{im}(\cdot)$ is the continuation value for the player given current states and actions. In practice we use the *ex-ante* value function. Its derivation is in the appendix. The function $\sigma_{im}(\cdot|\mathbf{s_m^t})$ accounts for i's beliefs on -i's actions given current states.

Firstly, players' best response functions are characterized. We define: $\Pi_{im}(\mathbf{a_m^t}, \mathbf{s_m^t}) = \widetilde{\pi}_{im}(\mathbf{a_m^t}, \mathbf{s_m^t}) + \mathbf{1}(a_{im}^t = 1) \cdot \xi_{im}^t$, where $\widetilde{\pi}_i(\mathbf{a_t}, \mathbf{s_m^t})$ comprises all the terms in (2) but the iid

part of the profitability shock and, $\widetilde{\Pi}_{im}(\mathbf{a}_{\mathbf{m}}^{\mathbf{t}}, \mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) = \widetilde{\pi}_{im}(\mathbf{a}_{\mathbf{m}}^{\mathbf{t}}, \mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) + \beta \cdot \sum_{\mathbf{s}_{\mathbf{m}}^{\mathbf{t}+1}} p_m^s(\mathbf{s}_{\mathbf{m}}^{\mathbf{t}+1} | \mathbf{s}_{\mathbf{m}}^{\mathbf{t}}, \mathbf{a}_{\mathbf{m}}^{\mathbf{t}}) \cdot V_{im}(\mathbf{s}_{\mathbf{m}}^{\mathbf{t}+1}; \sigma_{im}(\cdot)).$ Then the *i*'s best response is implicitly defined by²⁴:

$$H_{im}(a_{i}^{t} = 1 | \mathbf{s}_{\mathbf{m}}^{t}; \sigma_{im}(\cdot)) =$$

$$1 - exp \left\{ -exp \left\{ \begin{array}{l} \sum_{\mathbf{a}_{-im}^{t}} \sigma_{im}(\mathbf{a}_{-im}^{t} | \mathbf{s}_{\mathbf{m}}^{t}) \cdot \widetilde{\Pi}_{im}(a_{im}^{t} = 1, \mathbf{a}_{-im}^{t}, \mathbf{s}_{\mathbf{m}}^{t}) - \\ \sum_{\mathbf{a}_{-im}^{t}} \sigma_{im}(\mathbf{a}_{-im}^{t} | \mathbf{s}_{\mathbf{m}}^{t}) \cdot \widetilde{\Pi}_{im}(a_{im}^{t} = 0, \mathbf{a}_{-im}^{t}, \mathbf{s}_{\mathbf{m}}^{t}) \end{array} \right\} \right\}$$
(5)

The solution to this problem is a vector of player i's optimal actions when he faces each possible configuration for the state vector $\mathbf{s_m^t}$ and has consistent beliefs about other players actions in the same states of the world.

By stacking up best responses for every player and every state one can form a system of $1 \times N_s$ equations, where N_s expresses the number of different possible states in this market. This system is used to find the $1 \times N_s$ vector of players' beliefs. A formal proof of the existence of this vector can be found in Pesendorfer and Schmidt-Dengler (2008). Uniqueness of this equilibrium is not, however, guaranteed. This is a common feature of entry games. The estimation procedure is designed to deal with the multiplicity of equilibria.

5 Econometric Model

This section describes identification and the estimation procedure. We show that the estimator developed in Pesendorfer and Schmidt-Dengler (2008) can be reduced to a simple OLS estimator with a closed form solution. Our representation of the problem avoids the use of numerical methods in the estimation procedure and reduces significantly the computational burden.

5.1 Identification

Following the CCP approach (Hotz and Miller (1993)) we firstly identify the vector of entry probabilities for public and private players and the transitions directly from the data. For the identification of entry probabilities we need to introduce two assumptions:

Assumption (i): There are no unobserved common knowledge states.

Assumption (ii): The same equilibrium is played in all available markets.

 $^{^{24}\}mathrm{For}$ a proof see the appendix.

These identifying assumptions follow Ryan (2012). Pesendorfer (2010), Aguirregabiria and Nevo (2010) and Bajari, Hong and Nekipelov (2010) discuss the importance of the assumptions above.

Arcidiacono and Miller (2011) relaxes assumption (i). Our reduced form evidence suggests that our results are robust to the inclusion of market, time and player level unobservables. This mitigates our concern with unobservables. Regarding assumption (ii), we could deal with the multiplicity problem by estimating the model for each market we observe in our sample, as proposed by Pesendorfer and Schmidt-Dengler (2008). The main problem is that because most of these markets are quite stable, i.e. we do not observe frequent entry and exit movements, we could not accurately identify the reduced form parameters and, therefore, the structural parameters associated to most of our markets. In our application it is necessary to pool the data of different markets. The same is done in Collard-Wexler (2010) and Ryan (2012).

Under these assumptions the identification of Θ_{im} follows from Pesendorfer and Schmidt-Dengler (2008).

5.2 An OLS estimator for the structural parameters

We use the idea proposed by Pesendorfer and Schmidt-Dengler (2003, 2008) to recover the structural parameters, Θ . This consists in minimizing the following weighted distance:

$$\min_{\Theta} \left[\hat{H} - f\left(\mathbf{s}; \hat{\sigma}(\cdot), \hat{p}^{s}(\cdot), \Theta\right) \right]' \hat{W} \left[\hat{H} - f\left(\mathbf{s}; \hat{\sigma}(\cdot), \hat{p}^{s}(\cdot), \Theta\right) \right]$$
(6)

Here \hat{H} is a $N_s \times 1$ vector of consistent estimates of $H_{im}(a_i^t = 1 | \mathbf{s}_m^t; \sigma_{im}(\cdot))$ for each player/market in each possible state and is estimated in the first stage; $f(\cdot)$ gives us the RHS of (5) for each player/market in each possible state and depends on consistent estimates of beliefs, $\hat{\sigma}(\cdot)$, and transitions, $\hat{p}^s(\cdot)$. Beliefs and transitions are also estimated in the first stage. \hat{W} is a positive definite $N_s \times N_s$ matrix that weights the difference between the LHS and RHS of (5) in each state and depends on states and the beliefs and transitions estimated in the first stage.

In practice, the estimation based on formulation (6) can be very time consuming depending on the number of states and parameters. It involves a complex non linear search of a multidimensional vector of parameters minimizing the weighted quadratic sum in the minimand of the problem above. When N_s is large, as in our application, this procedure can take considerable time. By using the OLS approach described in this section we calculate $\hat{H} - f(\mathbf{s}; \hat{\sigma}(\cdot), \hat{p}^s(\cdot), \Theta)$ only once. Monte Carlo experiments in Sanches and Silva (2012) show that this approach is 90 times faster than the ALSE-I²⁵ in a entry game with two players.

The equivalence between the ALSE-I and OLS is stated the next proposition. The proposition follows from the linearity of the best response function in Θ which, in turn, follows from the linearity of the value function in Θ . These intermediate results are shown in the appendix.

Proposition 1. Given the linearity of the best response function (5) and the ALSE regularity conditions in Gourieroux and Monfort (1995) the ALSE-I reduces to the OLS estimator. Moreover, when $n_{fs} \to \infty$, where n_{fs} is the number of observations used to compute the beliefs and transitions in the first stage, its asymptotic distribution is given by $\sqrt{n} \left(\hat{\Theta} - \Theta \right) \sim N(\mathbf{0}, \Xi)$, where:

- 1. $\Xi = (M'M)^{-1}M'G\Omega G'M(M'M)^{-1}$ is the asymptotic variance-covariance matrix of the OLS estimator;
- 2. $G = \nabla_{(p,\sigma)'} [(y^* M \cdot \Theta)]$, the derivative of the best response function w.r.t the auxiliary parameters, (p, σ) ;
- 3. *M* is a $N_s \times N_p$ matrix. N_p equal to the number of parameters in Θ ; this matrix depends on states and auxiliary parameters and is defined in the appendix; and,

4. Ω is the variance-covariance matrix of the auxiliary parameters.

Proof. See the appendix.

To compute the variance we need just to calculate $\nabla_{(p,\sigma)'} [(y^* - M \cdot \Theta)]$, the derivative of $y^* - M \cdot \Theta$ with respect to the auxiliary (first stage) parameters, and Ω . To calculate $\nabla_{(p,\sigma)'} [(y^* - M \cdot \Theta)]$ we perturb each estimated auxiliary parameter and compute the change in $y^* - M \cdot \Theta$ for every state. To compute Ω we bootstrap a sequence of auxiliary parameters from the original sample and use the sample analogue of Ω to get a consistent estimate of this element.

Although intuitive, the result in proposition 1 is interesting and can be extended in several ways. It shows that with consistent first stage estimates for the auxiliary parameters

²⁵Asymptotic Least Squares Estimator when the matrix of weights is equal to the identity matrix.

OLS delivers consistent estimates of Θ . The time for the estimation is severely reduced. A standard econometric package such as STATA is enough to solve the problem²⁶.

Computationally, for larger state spaces the *ex ante* value function may be infeasible because it depends on the inversion of a $N_s \times N_s$ matrix, where N_s is the number of states in the model. When this is the case, researchers may use forward simulation to estimate the value function as is shown in Hotz, Miller, Sanders and Smith (1994). It is readily seen that the value function will continue to be linear and the procedure above can be applied. Our procedure can reduce significantly the time involved in the estimation even in these situations.

Our approach can be easily extended to other weight matrices. As in Gourieroux and Monfort (2005), one can derive the optimal weight matrix and use it instead of the identity to solve (6). In this case our proposition imply that the ALSE still has a closed form solution that is equal to the traditional GMM formula. These extensions and the small sample properties of the estimations are analyzed in Sanches and Silva-Junior (2012). For a small first stage sample the efficient ALSE (with the optimal weight matrix) can be used to reduce the small sample bias of the ALSE-I estimator. In our application, as the number of observations in the first stage is large, this small sample problem is of second order. We find that the OLS estimator does a good job - see Pesendorfer and Schmidt-Dengler (2008).

5.3 CCPs and state space

Following the CCP approach the empirical implementation of the model depends on (i) the estimation of beliefs and actions for each player, respectively, $H_{im}(a_i^t = 1 | \mathbf{s}_{\mathbf{m}}^t; \sigma_{im}(\cdot))$ and $\sigma_{im}(\mathbf{a}_{-i\mathbf{m}}^t | \mathbf{s}_{\mathbf{m}}^t)$ and (ii) the estimation of a transition process for the exogenous states, $p_m^s(\cdot)$. Next our estimation procedure for these elements is discussed.

5.3.1 Reduced form estimation of beliefs

We estimated equation (1) pooling the two private players, Bradesco and Itau. The data include the markets where Bradesco was active for at least one period and the markets where Itau was active for at least one period²⁷.

Instead of including year dummies we included $\overline{x}_t = \sum_m x_{mt}$, the total payroll of the municipalities in our sample in a given year, to control for the correlation in the decisions of

²⁶The same approach could also be easily applied for the estimation of static games of incomplete information (see Bajari, Hong, Krainer and Nekipelov (2009)).

 $^{^{27}\}mathrm{This}$ follows the definition of potential markets defined in section 3.

private players in the same period of time. Instead of including state dummies we constructed 4 categories of markets. This keeps the state space of the structural model reduced. The market categories are defined according to the number of potential competitors in a given market ²⁸. More specifically, if N_m is the number of potential competitors in municipality m then $M_{1m} = 1$ if $N_m \leq 2$; $M_{2m} = 1$ if $2 < N_m \leq 4$; $M_{3m} = 1$ if $4 < N_m \leq 6$; and $M_{4m} = 1$ if $N_m \geq 7$. With this definition we can substitute $\mu_m = \sum_{k=1}^{4} \gamma_k M_{km}$. The same strategy was used in Collard-Wexler (2010). The vector $\mathbf{x_{mt}}$ includes only municipality payroll.

Entry decisions for public players were estimated using the same specification but excluding n_{mt-1}^{pub} and n_{mt-1}^{pri} from the set of covariates. We pooled the two public players, Bank of Brazil and Caixa Economica Federal. The data include the markets where Bank of Brazil was active for at least one period and the markets where Caixa Economica Federal was active for at least one period.

We estimated the logits for the samples 1988-2010 and 1996-2010. The sample 1996-2010 excludes the hyperinflation period and allows us to focus on the more recent market trends. The coefficients in the logits for public/private players are in the appendix.

5.3.2 State space and transitions for exogenous states

Two different identification strategies to estimate the structural model are exploited. The first strategy excludes time and market effects as in Ryan (2012). The advantage of this formulation is that it uses a reduced state space. The second formulation uses market dummies and the sample payroll to control for market and time effects. Both strategies are based on the empirical CCP estimates. Only the structural parameters for the private players, Bradesco and Itau, are estimated.

Strategy 1: Model without time and market effects. The state space for any private player, $i \in i_{pri}$, is composed by the following elements:

$$\mathbf{s_i^t} \in \left\{ a_i^{t-1}, \left\{ \mathbf{a_j^{t-1}} \right\}_{j \neq i}, x^t, \left\{ \mathbf{I}(i=k) \right\}_{k \in i_{pri}} \right\}$$

Here $\{\mathbf{I}(i=k)\}_{k\in i_{pri}}$ is a set of private players dummies and x^t is the municipality payroll. The other elements are the actions of player *i* in period t-1, a_i^{t-1} , and the actions of player *i*'s competitors in period t-1, $\{\mathbf{a}_j^{t-1}\}_{i\neq i}$. The variable x^t is discretized in 10 deciles.

The law of motion for x^t is estimated by a simple auto-regressive ordered logit. This formulation for the law of motion of x^t ignores potential effects of banks, either public or

 $^{^{28}}$ Our definition of potential competitor is based on the super efficient estimator in the section 3 - i.e. the number of potential competitors in municipality m is equal to the maximum number of players that were active for at least one period in municipality m since 1900.

private, on municipality income.

The state space of this model is composed by $2 \cdot 2^3 \cdot 10 \cdot 2 = 320$ elements.

■ Strategy 2: Model with time and market effects. The second model includes time and market effects. Market effects are captured by 4 market dummies. Time effects are captured by the sample payroll. Market dummies and the sample payroll variables were defined above - see section 5.3.1. The state space in this model is:

$$\mathbf{s_{im}^{t}} \in \left\{ a_{im}^{t-1}, \left\{ \mathbf{a_{jm}^{t-1}} \right\}_{j \neq i}, x_{m}^{t}, \bar{x}^{t}, \left\{ \mathbf{I}(i=k) \right\}_{k \in i_{pri}}, \left\{ \mathbf{I}(m=k) \right\}_{k=1}^{4} \right\}$$

Here $\{\mathbf{I}(m=k)\}_{k=1}^{4}$ is a set of market dummies for the 4 market types; $\bar{x}^{t} = \sum_{m=1}^{4} w_{m} x_{m}^{t}$ is the sample payroll, where, w_{m} is the number of markets of type m and x_{m}^{t} is the average payroll of type m markets in period t; a_{im}^{t-1} is player i's action in a market of type m in t-1, $\{\mathbf{a}_{jm}^{t-1}\}_{j\neq i}$ are the actions of player i's competitors in the same market in period t-1 and $\{\mathbf{I}(i=k)\}_{k\in i_{mri}}$ is a set of dummies for each private player.

The law of motion for x_m^t is calculated using an auto-regressive ordered logit structure. The variable x_m^t is discretized in four percentiles for each market type. A model for each market type was estimated. Finally \bar{x}^t was calculated using $\bar{x}^t = \sum_{m=1}^4 w_m x_m^t$ under the assumption that w_m is fixed over time.

The estimation of the model with market dummies is time consuming because the inclusion of the sample payroll, which depends on the realization of the payroll variable in every market, increases exponentially the dimension of the state space. The state space has $2 \cdot 2^3 \cdot 4^4 \cdot 2 \cdot 4 = 32768$ elements.

5.4 Results

We assumed that β , the discount factor, is equal to 0.90. To focus on the more recent trends of the market we used the CCPs and transitions estimated with the 1996-2010 sample. The CCPs of the non strategic public players correspond to models I and II in the second block of Table 11 - those estimated using the sample 1996-2010. For the private players the CCPs are given by models I and II in the second block of Table 12. We used the OLS estimator to estimate the model.

Parameters are estimated in units of the scale factor in the EV distribution and do not have a level interpretation. Only relative magnitudes matter. Standard errors of the parameters were calculated by block bootstraping CCPs and transitions 100 times. The structural model was estimated 100 times, one for each block bootstrap draw of beliefs and transitions. The standard error across this set of parameters was calculated. A similar procedure was applied in Ryan (2012) and Collard-Wexler (2010).

Table 5 reports the structural parameters. The first column corresponds to the model without market unobservables. The second column shows the model with market dummies and the sample payroll, estimated according to strategy 2.

	(I)	(II)
	Profit Co	omponents
N Public	0.0605	0.0726
	[0.01]	[0.00]
N Private	-0.0487	-0.0256
	[0.01]	[0.00]
Market Payroll*	-0.0001	0.0019
	[0.00]	[0.00]
Constant	-0.3720	-0.5821
	[0.01]	[0.05]
	Shock Co	omponents
Sample Payroll*		0.0004
		[0.00]
Market 1		0.2377
		[0.04]
Market 2		0.2104
		[0.03]
Market 3		0.1176
		[0.02]
	Entry/Pl	ayer Costs
Entry Costs	-4.9272	-5.7442
	[0.09]	[0.02]
Dummy Bradesco	-0.0270	-0.0245
	[0.01]	[0.02]
Observations	320	32768
Note: (*) Sample pay	roll measure	d in R\$ bil-

 Table 5: Structural Parameters for Private Players

Note: (*) Sample payroll measured in R\$ billions of 2011; market payroll measured in R\$ millions of 2011. Standard-errors in brackets. Standard errors obtained from 100 block bootstraps of beliefs and transitions. Parameters are measured in units of standard deviations of the iid profitability shock.

Qualitatively, both specifications produce similar results. The main difference is that in model I, the market payroll coefficient is negative but small and not significant. All models predict that the entry of a new private competitor reduces the profits of the private incumbent. The entry of a new public player increases the profits of a private incumbent. The constant term, which measures operational costs, is negative and relatively larger in the second model. Entry costs are also negative and relatively larger in the second model. The contribution of the components of the shock in the second model is relatively important. The coefficient attached to the sample payroll is positive. This means that increases in the sample income shifts to the right the distribution of the shock and increases entry rates. Market effects are positive.

To facilitate the interpretation of these results, the next table reports the estimates as percentage of entry costs. This means that coefficients are divided by the absolute value of the entry costs.

		0
	(I)	(II)
	Profit Co	mponents
N Public	1.228%	1.264%
N Private	-0.988%	-0.446%
Market Payroll*	-0.002%	0.033%
Constant	-7.550%	-10.134%
	Shock Co	mponents
Sample Payroll*		0.007%
Market 1		4.138%
Market 2		3.663%
Market 3		2.047%
	Entry/Pla	ayer Costs
Entry Costs	100.000%	100.000%
Dummy Bradesco	-0.549%	-0.427%

Table 6: Structural Parameters as Percentage of the Entry Costs

Note: (*) Sample payroll measured in R\$ billions of 2011; market payroll measured in R\$ millions of 2011.

Again, the predictions of both models are quite close. In particular, entry of a new public player increases profits of the private incumbent in around 1.3% of the entry costs. Entry of a new private player reduces profits of the private incumbent in around 0.45-0.9% of the entry costs.

The next table provides profit estimates for the private banks using the structural parameters. These parameters allow us to estimate a measure of return over entry costs. We also simulate the number of years necessary to recover entry costs.

Table 7: Average Period Profits and Return to Fixed Costs in Private Monopoly Markets

	(I)	(II)
Period Profits in Std Deviations	0.1936	0.2493
Period Profits as $\%$ of Entry Costs	3.930%	4.339%
Years to Recover Entry Costs	26.0	23.0
Note: Average profits of a private monoper (market in the lower market payroll decile of entry costs corresponds to the period p divided by the entry cost. To calculate the recover the entry costs we assumed a disc the market payroll is increasing steadily a a monopoly structure every period.). Period pr rofit in std o ne number o ount rate of	ofits as % deviations of years to f 0.9, that

The average period payoff of the private banks in monopoly markets is computed in the the first line of the table. The results show that the second model predicts larger profits. The second line shows that model I predicts returns to the entry investment of 4% in monopoly markets. Model II predicts that returns over entry costs are slightly larger, around $4.4\%^{29}$.

The third line shows the number of years necessary to recover entry costs. We assumed $\beta = 0.9$ and that municipality and sample payroll are growing steadily at 3% per year. We accumulated the discounted payoffs and computed the number of years that are necessary to recover the estimated entry costs. Model I predicts that in monopoly markets it takes on average 26 years to the private player recover the entry investment. Model II predicts that a private player needs on average 23 years to cover the entry cost³⁰.

5.5 Discussion

Two remarkable facts arise from our analysis:

- 1. Public players complement private players;
- 2. Private players crowd-out other private players.

²⁹To calculate profits we fixed the sample payroll at its 2010 average value in smaller markets, that is in markets of type 1. For model II we assumed that the market dummy for markets of type 1 is equal to one. Thus the results are calculated for markets type 1. The sample payroll is used only to compute profits in model II and is equal to the sample payroll of 2010.

³⁰This means that entry barriers are quite high. A recent expansion plan of Bank of Brazil illustrates this point. BB set down R\$1 billion to construct 600 new branches in the Brazilian territory. This implies that on average each new branch costs R\$1.66 million.

Notice also that the potential demand of a small market is quite small: The average yearly payroll of a market in our sample was R\$9 million in 2010 and only a small fraction of the population demands banking services. In 2011 the Institute of Applied Economic Research (IPEA, 2011), an institute of the Brazilian federal government, estimated that around 40% of the Brazilian population has no access to any kind of banking services. This percentual can be even large in the markets represented in our sample.

The first result shows that profit of public banks are positively affected by the number of public branches in the same market. Our estimates do not allow us to disentangle the details of the spill-over channels. Broadly speaking, our findings are consistent with public banks (i) having monopoly over a number of important Federal funds and (ii) being driven by social, as opposed to strategic or market reasons. The first element guarantees a large volume of credit for small markets - see Feler (2012). The second induces product differentiation between public and private banks: Public and private banks target different clients - see Coelho, Melo and Rezende (2012). In this case, the amount of cheap credit and public transfers poured by the public banks in small isolated municipalities shifts the demand for banking services, making these markets more attractive for private players. This effect induces the entry of private players.

The crowd-out effect is consistent with Coelho et al (2012). They found that the competition between the Brazilian private banks is relatively tough. Our results reinforce the view that private banks are competitive.

6 Model Fit and Counterfactuals

This section uses the structural model to construct a policy experiment. We are interested in the following question: What happens with the supply of private financial services in small isolated markets when public banks are privatized?

First we solve the model using the estimated parameters. The solution to the model is a vector of N_s entry probabilities that solves the system of implicit best responses given by equation (5).

For models with a large state space this exercise is not computationally feasible. The state space of model II has dimension $N_s = 32768$. The solution for this model showed to be beyond our computational possibilities. The time to solve the model increases exponentially with the state space. From now on, we use only model I, that has a reduced state space $(N_s = 320)$, to compute the counterfactual experiments.

6.1 Model fit

We solved the system (5) for private banks entry probabilities. This system is non linear. This means that its solution is not necessarily unique.

To check how the multiplicity affects our conclusions, we proceed in the following way:

First, we solve model I for the entry probabilities using the logit probabilities as the initial guess; second, we perturbed the logit probabilities; third we computed again the solution for the model using the "perturbed" vector of logit probabilities as the initial guess; fourth, we compared the "perturbed" solution with the original solution³¹. Doing so we find that the solutions were identical for any initial guess.

We compare the solution obtained from the structural model with the logit probabilities for all available states. The next table illustrates some statistics of our predictions.

	(I)
Correlation Fitted and Logit Probabilities	99.91%
Average Sum of Squared Errors	0.05%
Average Sum of Errors	0.82%
Note: Correlation between the probabilities obtained solution of model I and the logit model (model I, T	
1996-2010) for each state (320 states). Average sum errors gives the sum of the squared difference between	

and the model probabilities for each state averaged across states. The average sum of errors gives the sum of the differences between the logit and the model probabilities averaged across the

320 states.

Table 8: Fitted vs Sample (Logit) Probabilities

In the first line the correlation between the logit probabilities and the solution of the structural model for all states is calculated. The second shows the average squared difference between the logit and the structural probabilities. The third line shows the average sum of these differences. The fitting of the model is very good. The correlation between the logit and structural probabilities is high. The average error of the structural probabilities is below 1%.

We performed an additional exercise. We took the smallest market in terms of sample payroll and assumed that in the first period all the four banks are out of the market. We used firstly the probabilities predicted by the logit models and simulated 1000 paths 100 periods ahead of private banks actions and then we constructed an average path taking the mean across the 1000 paths. We did the same using the probabilities predicted by the structural models. The next figure compares the paths implied by the logit and by the structural model.

 $^{^{31}}$ Firstly we multiplied the original guesses (calculated from the CCPs showed above) by several factors between 0 and 1. We also started the model with a "fixed" guess, where the probabilities for all the states and for all the players are equal to 0.25, 0.5 and 0.75. We used the same procedure to compute the counterfactuals.

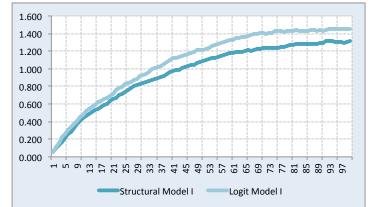


Figure 2: Number of Private Banks 100 Periods Ahead - Model I

Note: Number of private banks in a small market starting from a state where all the competitors are out of the market. Paths 100 periods ahead simulated 1000 times using the structural and the logit probabilities for model I. The figure shows number of private branches averaged over 1000 simulations.

The figure shows that the path obtained from the structural model is close to the path obtained from the reduced form logit models in the first 30 periods. Subsequently the path of the structural model is below the path produced by the logit model. The structural model predicts that after 100 years this small market, without any public/private branch in operation in the first period, will have on average 1.32 private branches. The logit predicts that the same market will have 1.45 branches.

Next we use this model to construct counterfactuals.

6.2 Counterfactual: Privatization of public banks

This section analyzes the effects of the privatization of both public banks on the total supply of financial services in small isolated markets. We assumed that each public bank is bought by different players: BB is bought by one player and CEF by the other. We assumed that the coefficient attached to the number of public competitors in the structural model is equal to the coefficient attached to the number of private competitors. The entry probabilities of public players, instead of being generated by an exogenous process, are calculated according to the system of best responses showed in equation (5).

We calculated the equilibrium probabilities for 4 players. Now this depends on the solution of a system of 640 equations and 640 unknown variables. To check how multiplicity affects our conclusions we used the procedure described in Section 6.1. In all experiments the resulting equilibrium did not change. These probabilities are used to simulate 1000 paths 100 periods ahead. The next figure shows the path for the total number of branches, public

plus private, after and before the privatization. We computed this path for a small market where the initial state is characterized by zero active players.

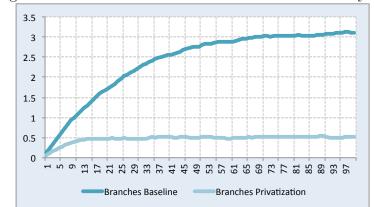


Figure 3: Counterfactual: Privatization of Public Players

Note: Number of branches (public plus private) in a small market starting from a state where all the competitors are out of the market (baseline and privatization counterfactual). Paths 100 periods ahead simulated 1000 times using the structural probabilities for model I. The figure shows number of private branches averaged over 1000 simulations. Branches privatization shows the total number of branches if public branches are privatized. Branches baseline is the total number of branches (public plus private) using the structural model I for private players and the non strategic behavior assumption for public players (calculated based on the logits in Table 11, model I, sample 1996-2010).

The exercise shows that in the long-run the total number of active branches in small municipalities drops from 3 to 0.5 on average. This means that with the privatization around 50% of the Brazilian small municipalities would not be attended by any bank branch. To assure that all these small municipalities would have at least one bank branch in the counterfactual world where public banks are bought by strategic players the government should give a subsidy of 8% over the operational costs of all active branches in the market. Using the fact that the structural model predicts that operational costs are around 7.55% of entry costs and an estimate of R\$1 million for the entry costs we calculated that the present cost of this policy is around R\$349,463.51 per municipality³². This value is relatively small compared to the market value of Brazilian public banks. BB's market value in 2012 was approximately US\$88 billions. This means that the resources raised with the privatization of BB would be sufficient to cover the subsidies for 240 thousand branches in the country or, approximately, 42 branches per municipality.

 $^{^{32}\}mathrm{Present}$ values for a time horizon of 100 years and using a discount factor of 0.9 per year.

7 Conclusions

This paper explores microdata of 1002 isolated markets in Brazil during 1988-2010 to estimate a dynamic entry game for public and private banks. We compute the market equilibrium in different counterfactual scenarios. Our setup allow us to make consistent *ex ante* analyses on the effects of changes in the banking market structure on variables of interest.

We obtained two main conclusions. First, public banks generate positive profit spill-overs for private banks; second, private banks crowd-out private competitors. Our estimates show that the entry of a public bank in a given market increases the return of a private incumbent by 1.2 percent and the entry of a private bank reduces the return of a private incumbent by 0.05 to 1 percent.

The counterfactual in which public banks are sold to private players shows that the total number of active branches operating in the long-run in a typical small market drops from 3 to 0.5 on average. To guarantee that, after the privatization, all small municipalities would have at least one active branch the government should give a subsidy of approximately US\$175,000 for each small market. This value is relatively small compared to the market value of Brazilian public banks. BB's market value in 2012 was approximately US\$88 billions. This means that the resources raised with the privatization of BB would be sufficient to cover the subsidies for 240 thousand branches in the country or, approximately, 42 branches per municipality.

Our estimation procedure improves Pesendorfer and Schmidt-Dengler (2008). We show that the OLS estimator gives us consistent estimates of the structural parameters. This approach allows us to avoid the use of numerical methods in the estimation of the structural parameters. Doing so we reduce significantly the computational burden.

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Appendix 1: Proofs

• Model solution. We omit the term Θ from the equations to simplify the notation and write $V(\mathbf{s_m^{t+1}}; \sigma_{im}(\cdot), \Theta_{im}) = V_{im}(\mathbf{s_m^{t+1}}; \sigma_{im}(\cdot))$ and $\Pi(\mathbf{a_m^t}, \mathbf{s_m^t}; \Theta_{im}) = \Pi_{im}(\mathbf{a_m^t}, \mathbf{s_m^t})$. We further simplify the notation as follows.

- 1. Isolate the iid shock from (1) writing $\Pi_{im}(\mathbf{a}_{\mathbf{m}}^{\mathbf{t}}, \mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) = \widetilde{\pi}_{im}(\mathbf{a}_{\mathbf{m}}^{\mathbf{t}}, \mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) + \mathbf{1}(a_{im}^{t} = 1) \cdot \xi_{im}^{t}$, where $\widetilde{\pi}_{i}(\mathbf{a}_{\mathbf{t}}, \mathbf{s}_{\mathbf{m}}^{\mathbf{t}})$ comprises all the terms in (2) but the iid part of the profitability shock.
- 2. With this we can write the deterministic part of the objective function simply as:

$$\widetilde{\Pi}_{im}(\mathbf{a}_{\mathbf{m}}^{\mathbf{t}}, \mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) = \widetilde{\pi}_{im}(\mathbf{a}_{\mathbf{m}}^{\mathbf{t}}, \mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) + \beta \cdot \sum_{\mathbf{s}_{\mathbf{m}}^{\mathbf{t}+1}} p_m^s(\mathbf{s}_{\mathbf{m}}^{\mathbf{t}+1} | \mathbf{s}_{\mathbf{m}}^{\mathbf{t}}, \mathbf{a}_{\mathbf{m}}^{\mathbf{t}}) \cdot V_{im}(\mathbf{s}_{\mathbf{m}}^{\mathbf{t}+1}; \sigma_{im}(\cdot))$$
(7)

3. Using these facts rewrite i's problem in the following way:

$$\underset{a_{i}^{t} \in \{0,1\}}{\operatorname{Max}} \sum_{\mathbf{a}_{-\mathrm{im}}^{t}} \sigma_{im}(\mathbf{a}_{-\mathbf{i}}^{t} | \mathbf{s}_{t}) \left\{ \widetilde{\Pi}_{im}(\mathbf{a}_{\mathbf{m}}^{t}, \mathbf{s}_{\mathbf{m}}^{t}) + \mathbf{1}(a_{i}^{t} = 1) \cdot \xi_{im}^{t} \right\}$$
(8)

Now, we analyze i's decision process. Given beliefs and states the expected payoff derived from $a_{im}^t = 1$ and $a_{im}^t = 0$ is respectively:

$$E\Pi_{im}(a_{im}^{t} = 1 | \mathbf{s}_{\mathbf{m}}^{\mathbf{t}}; \sigma_{im}(\cdot)) =$$

$$\xi_{im}^{t} + \sum_{\mathbf{a}_{-im}^{\mathbf{t}}} \sigma_{im}(\mathbf{a}_{-im}^{\mathbf{t}} | \mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) \cdot \widetilde{\Pi}_{im}(a_{im}^{t} = 1, \mathbf{a}_{-im}^{\mathbf{t}}, \mathbf{s}_{\mathbf{m}}^{\mathbf{t}})$$

and,

$$E\Pi_{im}(a_{im}^t = 0 | \mathbf{s}_{\mathbf{m}}^{\mathbf{t}}; \sigma_{im}(\cdot)) = \sum_{\mathbf{a}_{-im}^{\mathbf{t}}} \sigma_{im}(\mathbf{a}_{-im}^{\mathbf{t}} | \mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) \cdot \widetilde{\Pi}_{im}(a_{im}^t = 0, \mathbf{a}_{-im}^{\mathbf{t}}, \mathbf{s}_{\mathbf{m}}^{\mathbf{t}})$$

Therefore, given states and beliefs, $a_{im}^t = 1$ iff:

$$\xi_{im}^{t} + \sum_{\mathbf{a}_{-im}^{t}} \sigma_{im}(\mathbf{a}_{-im}^{t} | \mathbf{s}_{m}^{t}) \cdot \widetilde{\Pi}_{im}(a_{im}^{t} = 1, \mathbf{a}_{-im}^{t}, \mathbf{s}_{m}^{t})$$

$$\geq \sum_{\mathbf{a}_{-im}^{t}} \sigma_{im}(\mathbf{a}_{-im}^{t} | \mathbf{s}_{m}^{t}) \cdot \widetilde{\Pi}_{im}(a_{im}^{t} = 0, \mathbf{a}_{-im}^{t}, \mathbf{s}_{m}^{t})$$

Now using $\xi_{im}^t \sim EV(0,1)$ we can find a closed form solution for i's best response function given states and beliefs:

$$H_{im}(a_i^t = 1 | \mathbf{s}_{\mathbf{m}}^{\mathbf{t}}; \sigma_{im}(\cdot)) =$$

$$1 - exp \left\{ -exp \left\{ \begin{array}{l} \sum_{\mathbf{a}_{-im}^{\mathbf{t}}} \sigma_{im}(\mathbf{a}_{-im}^{\mathbf{t}} | \mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) \cdot \widetilde{\Pi}_{im}(a_{im}^t = 1, \mathbf{a}_{-im}^{\mathbf{t}}, \mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) - \\ \sum_{\mathbf{a}_{-im}^{\mathbf{t}}} \sigma_{im}(\mathbf{a}_{-im}^{\mathbf{t}} | \mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) \cdot \widetilde{\Pi}_{im}(a_{im}^t = 0, \mathbf{a}_{-im}^{\mathbf{t}}, \mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) \end{array} \right\} \right\}$$

■ Value function. Following Pesendorfer and Schmidt-Dengler (2008) we define the value function as the continuation value for profits before shocks are observed and actions are taken. Mathematically:

$$V_{im}(\mathbf{s}_{\mathbf{m}}^{\mathbf{t}};\sigma_{i}(\cdot)) = \sigma_{im}(a_{im}^{t}=1|\mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) \cdot E\left[\xi_{im}^{t}|a_{im}^{t}=1,\mathbf{s}_{\mathbf{m}}^{\mathbf{t}}\right] + \sum_{\substack{a_{im}^{t},\mathbf{a}_{-\mathbf{im}}^{\mathbf{t}}}} \sigma_{im}(a_{im}^{t},\mathbf{a}_{-\mathbf{im}}^{\mathbf{t}}|\mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) \cdot \widetilde{\Pi}_{im}(\mathbf{a}_{\mathbf{m}}^{\mathbf{t}},\mathbf{s}_{\mathbf{m}}^{\mathbf{t}})$$
(9)

In this formulation $E[\xi_{im}^t | a_{im}^t = 1, \mathbf{s}_{\mathbf{m}}^t]$ is the (conditional) expectation operator with respect to the iid EV payoff shock and $\sigma_{im}(a_{im}^t, \mathbf{a}_{-im}^t | \mathbf{s}_{\mathbf{t}}^{\mathbf{m}})$ is the joint probability distribution of players' actions given states.

Now if we stack up (9) for every one of the N_s states of the we find a system of N_s equations in N_s unknown variables, in this case the value function for each state. The solution to this (linear) system provides us with a unique continuation value for each state as a function of parameters, beliefs and the distribution of future states. The solution for any state $\mathbf{s_m^t}$ can be easily plugged in equation (5), leaving us with a (non-linear) system of N_s equations with N_s unknown beliefs.

Before proceeding we are going to insert (7) into (9):

$$V_{im}(\mathbf{s}_{\mathbf{m}}^{\mathbf{t}};\sigma_{i}(\cdot)) = \sigma_{im}(a_{im}^{t}=1|\mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) \cdot E\left[\xi_{im}^{t}|a_{im}^{t}=1,\mathbf{s}_{\mathbf{m}}^{\mathbf{t}}\right] + \sum_{\mathbf{a}_{im}^{\mathbf{t}}}\sigma_{im}(\mathbf{a}_{im}^{\mathbf{t}}|\mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) + \beta \cdot \sum_{\mathbf{s}_{\mathbf{m}}^{\mathbf{t}+1}} p_{m}^{s}(\mathbf{s}_{\mathbf{m}}^{\mathbf{t}+1}|\mathbf{s}_{\mathbf{m}}^{\mathbf{t}},\mathbf{a}_{\mathbf{m}}^{\mathbf{t}}) \cdot V_{im}(\mathbf{s}_{\mathbf{m}}^{\mathbf{t}+1};\sigma_{im}(\cdot))\right\}$$

 $\begin{aligned} & \text{Call} \sum_{\mathbf{a}_{im}^{\mathbf{t}}} \sigma_{im}(\mathbf{a}_{im}^{\mathbf{t}}|\mathbf{s}_{m}^{\mathbf{t}}) \cdot \widetilde{\pi}_{im}(\mathbf{a}_{m}^{\mathbf{t}},\mathbf{s}_{m}^{\mathbf{t}}) = \mathbf{\Sigma}_{im}(\mathbf{a}_{m}^{\mathbf{t}}|\mathbf{s}_{m}^{\mathbf{t}}) \cdot \mathbf{\Gamma}_{im}(\mathbf{a}_{m}^{\mathbf{t}},\mathbf{s}_{m}^{\mathbf{t}}), \text{ where } \mathbf{\Sigma}_{im}(\mathbf{a}_{m}^{\mathbf{t}}|\mathbf{s}_{m}^{\mathbf{t}}) \text{ is a} \\ & 1 \times 2^{N} \text{ vector of probabilities for all possible tuple of players actions given states, } \mathbf{\Gamma}_{im}(\mathbf{a}_{m}^{\mathbf{t}},\mathbf{s}_{m}^{\mathbf{t}}) \\ & \text{ is the } 2^{N} \times 1 \text{ vector of (non stochastic) profits in that given state for all possible combina$ $tion of players actions and <math>\sum_{\mathbf{a}_{im}^{\mathbf{t}}} \sum_{\mathbf{s}_{m}^{\mathbf{t}+1}} \sigma_{im}(\mathbf{a}_{im}^{\mathbf{t}}|\mathbf{s}_{m}^{\mathbf{t}}) \cdot p_{m}^{s}(\mathbf{s}_{m}^{\mathbf{t}+1}|\mathbf{s}_{m}^{\mathbf{t}},\mathbf{a}_{m}^{\mathbf{t}}) = \mathbf{Z}_{m}(\mathbf{s}_{m}^{\mathbf{t}+1}|\mathbf{s}_{m}^{\mathbf{t}}), \text{ where } \\ & \mathbf{Z}_{m}(\mathbf{s}_{m}^{\mathbf{t}+1}|\mathbf{s}_{m}^{\mathbf{t}}) \text{ is a } 1 \times N_{s} \text{ vector containing the (weighted) probabilities of each state when the states are } \\ & \text{states are } \mathbf{s}_{m}^{\mathbf{t}} \text{ and } \Psi_{i}(\mathbf{s}_{m}^{\mathbf{t}}) \equiv \sigma_{im}(a_{i}^{t}=1|\mathbf{s}_{m}^{\mathbf{t}}) \cdot E\left[\xi_{im}^{t}|a_{im}^{t}=1,\mathbf{s}_{m}^{\mathbf{t}}\right]. \text{ Using this we can rewrite the equation above as:} \end{aligned}$

$$V_{im}(\mathbf{s}_{\mathbf{m}}^{\mathbf{t}};\sigma_{im}(\cdot)) =$$

$$\Psi_{\mathbf{i}}(\mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) + \Sigma_{\mathbf{im}}(\mathbf{a}_{\mathbf{m}}^{\mathbf{t}}|\mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) \cdot \Gamma_{\mathbf{im}}(\mathbf{a}_{\mathbf{m}}^{\mathbf{t}},\mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) + \beta \cdot \mathbf{Z}_{\mathbf{m}}(\mathbf{s}_{\mathbf{m}}^{\mathbf{t}+1}|\mathbf{a}_{\mathbf{m}}^{\mathbf{t}},\mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) \cdot \mathbf{V}_{\mathbf{im}}(\mathbf{s}_{\mathbf{m}}^{\mathbf{t}+1};\sigma_{\mathbf{im}}(\cdot))$$

In this equation $\mathbf{V}_{im}(\sigma_{im}(\cdot))$ is a $N_s \times 1$ vector of value functions in which each line represents a value function for a particular state. Now, stacking up these equations for any possible state in the economy we can write:

$$\mathbf{V}_{im}(\sigma_{im}(\cdot)) = \mathbf{\Psi}_{im} + \mathbf{\Sigma}\mathbf{\Gamma}_{im} + \beta \cdot \mathbf{Z}_{m} \cdot \mathbf{V}_{im}(\sigma_{im}(\cdot))$$
(10)
= $[\mathbf{I} - \beta \cdot \mathbf{Z}_{m}]^{-1} \cdot [\mathbf{\Psi}_{m} + \mathbf{\Sigma}\mathbf{\Gamma}_{im}]$

The existence of the inverse matrix in the first bit of (10) follows from the dominant diagonal property - see Pesendorfer and Schmidt-Dengler (2008). Here Ψ_{im} is a $N_s \times 1$ vector stacking all possible values of $\Psi_i(\mathbf{s_m^t}) \equiv \sigma_{im}(a_i^t = 1 | \mathbf{s_m^t}) \cdot E[\xi_{im}^t | a_{im}^t = 1, \mathbf{s_m^t}], \Sigma \Gamma_{im}$ is a $N_s \times 1$ vector of expected (non stochastic) profits for each state (where the expectation is taken across each possible vector of actions conditional on states) and $\mathbf{Z_m}$ is a $N_s \times N_s$ transition matrix for all states.

• Linearity of the value function. Start from equation (10). The only term that depends on parameters is $\Sigma\Gamma_{im}$. Each line in the $N_s \times 1$ vector $\Sigma\Gamma_{im}$ (represented as $\Sigma_{im}(\mathbf{a}_m^t | \mathbf{s}_m^t)$. $\Gamma_{im}(\mathbf{a}_{\mathbf{m}}^{\mathbf{t}}, \mathbf{s}_{\mathbf{m}}^{\mathbf{t}}))$ corresponds to the expected (non stochastic) profits for each state (where the expectation is taken across each possible vector of actions conditional on states). Having in mind the parametrization in (2) and (3) it is straightforward to see that for any state $\mathbf{s}_{\mathbf{m}}^{\mathbf{t}}$, $\Sigma_{im}(\mathbf{a}_{\mathbf{m}}^{\mathbf{t}}|\mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) \cdot \Gamma_{im}(\mathbf{a}_{\mathbf{m}}^{\mathbf{t}}, \mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) = K(\mathbf{s}_{\mathbf{m}}^{\mathbf{t}})'\Theta_{im}$, where $K(\mathbf{s}_{\mathbf{m}}^{\mathbf{t}})'$ is a $1 \times N_p$ vector of variables that depends on the beliefs for all players and states, where N_p is the number of parameters in the model. Now, to simplify the notation, write $[\mathbf{I} - \beta \cdot \mathbf{Z}_{\mathbf{m}}]^{-1} = \Delta$, a $N_s \times N_s$ matrix that depends on the transition matrix $\mathbf{Z}_{\mathbf{m}}$. Going back to (10) and using these facts it is easy to see that $\mathbf{V}_{im}(\sigma_{im}(\cdot)) = \Delta \cdot \Psi_{im} + \Delta \cdot K \cdot \Theta_{im}$, where K is a $N_s \times N_p$ matrix of variables constructed by stacking up $K(\mathbf{s}_{\mathbf{m}}^{\mathbf{t}})'$ for all states.

• Linearity of the best response function. Start from equation (5). Use the definition of $\widetilde{\Pi}_{im}(\mathbf{a_m^t}, \mathbf{s_m^t})$ to write:

$$\begin{split} \sum_{\mathbf{a}_{-\mathrm{im}}^{\mathbf{t}}} \sigma_{im}(\mathbf{a}_{-\mathrm{im}}^{\mathbf{t}} | \mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) \cdot \widetilde{\Pi}_{im}(a_{im}^{t} = 0, \mathbf{a}_{-\mathrm{im}}^{\mathbf{t}}, \mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) = \\ \sum_{\mathbf{a}_{-\mathrm{im}}^{\mathbf{t}}} \sigma_{im}(\mathbf{a}_{-\mathrm{im}}^{\mathbf{t}} | \mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) \cdot \widetilde{\pi}_{im}(a_{im}^{t} = 0, \mathbf{a}_{-\mathrm{im}}^{\mathbf{t}}, \mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) + \\ \beta \cdot \sum_{\mathbf{a}_{-\mathrm{im}}^{\mathbf{t}}} \sum_{\mathbf{s}_{\mathbf{m}}^{\mathbf{t}+1}} \sigma_{im}(\mathbf{a}_{-\mathrm{im}}^{\mathbf{t}} | \mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) \cdot p_{m}^{s}(\mathbf{s}_{\mathbf{m}}^{\mathbf{t}+1} | \mathbf{s}_{\mathbf{m}}^{\mathbf{t}}, a_{im}^{t} = 0, \mathbf{a}_{-\mathrm{im}}^{\mathbf{t}}) \cdot V_{im}(\mathbf{s}_{\mathbf{m}}^{\mathbf{t}+1}; \sigma_{im}(\cdot)) \end{split}$$

Now we can use the equation above and rewrite the transition matrix (containing beliefs and transition between states) as $\sum_{\mathbf{a^t}_{-im}} \sum_{\mathbf{s^{t+1}_m}} \sigma_{im}(\mathbf{a^t}_{-im} | \mathbf{s^t_m}) \cdot p_m^s(\mathbf{s^{t+1}_m} | \mathbf{s^t_m}, a_{im}^t = 0, \mathbf{a^t}_{-im}) =$ $\mathbf{Z^0_m}(\mathbf{s^{t+1}_m} | \mathbf{s^t_m})$, where $\mathbf{Z^0_m}(\mathbf{s^{t+1}_m} | \mathbf{s^t_m})$ is $1 \times N_s$ vector containing the (weighted) probabilities of each state when the states are $\mathbf{s^t_m}$ and player *i* is choosing $a_{im}^t = 0$. Using the same argument that we used to state the linearity of the value function, it is easy to see that $\sum_{\mathbf{a^t}_{-im}} \sigma_{im}(\mathbf{a^t_{-im}} | \mathbf{s^t_m}) \cdot \widetilde{\pi}_{im}(a_{im}^t = 0, \mathbf{a^t}_{-im}, \mathbf{s^t_m}) = K(\mathbf{s^t_m}, a_{im}^t = 0)' \Theta_{im}$, where $K(\mathbf{s^t_m}, a_{im}^t = 0)'$ is a $1 \times N_p$ vector of variables when states are $\mathbf{s^t_m}$ and player *i* is choosing $a_{im}^t = 0$. Therefore we can write:

$$\sum_{\mathbf{a_{-im}^{t}}} \sigma_{im}(\mathbf{a_{-im}^{t}}|\mathbf{s_{m}^{t}}) \cdot \widetilde{\Pi}_{im}(a_{im}^{t} = 0, \mathbf{a_{-im}^{t}}, \mathbf{s_{m}^{t}}) = K(\mathbf{s_{m}^{t}}, a_{im}^{t} = 0)' \Theta_{im} + \beta \cdot \mathbf{Z_{m}^{0}}(\mathbf{s_{m}^{t+1}}|\mathbf{s_{m}^{t}}) \cdot V_{im}(\mathbf{s_{m}^{t+1}}; \sigma_{im}(\cdot))$$

By substituting the value function using $\mathbf{V}_{im}(\sigma_{im}(\cdot)) = \Delta \cdot \Psi_{im} + \Delta \cdot K \cdot \Theta_{im}$, it is easy to see that:

$$\sum_{\mathbf{a}_{-\mathbf{im}}^{\mathbf{t}}} \sigma_{im}(\mathbf{a}_{-\mathbf{im}}^{\mathbf{t}} | \mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) \cdot \widetilde{\Pi}_{im}(a_{im}^{t} = 0, \mathbf{a}_{-\mathbf{im}}^{\mathbf{t}}, \mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) = K(\mathbf{s}_{\mathbf{m}}^{\mathbf{t}}, a_{im}^{t} = 0)' \Theta_{im} + \beta \cdot \mathbf{Z}_{\mathbf{m}}^{\mathbf{0}}(\mathbf{s}_{\mathbf{m}}^{\mathbf{t}+1} | \mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) \cdot [\Delta \cdot \boldsymbol{\Psi}_{\mathbf{im}} + \Delta \cdot K \cdot \Theta_{im}]$$

Analogously, we can write $\mathbf{Z}_{\mathbf{m}}^{\mathbf{1}}(\mathbf{s}_{\mathbf{m}}^{\mathbf{t}+1}|\mathbf{s}_{\mathbf{m}}^{\mathbf{t}})$ as the equivalent of $\mathbf{Z}_{\mathbf{m}}^{\mathbf{0}}(\mathbf{s}_{\mathbf{m}}^{\mathbf{t}+1}|\mathbf{s}_{\mathbf{m}}^{\mathbf{t}})$ when $a_{im}^{t} = 1$ and $K(\mathbf{s}_{\mathbf{m}}^{\mathbf{t}}, a_{im}^{t} = 1)$ as the equivalent $K(\mathbf{s}_{\mathbf{m}}^{\mathbf{t}}, a_{im}^{t} = 0)$ when the action is $a_{im}^{t} = 1$. With this we can calculate $\sum_{\mathbf{a}_{-\mathbf{im}}^{\mathbf{t}}} \sigma_{im}(\mathbf{a}_{-\mathbf{im}}^{\mathbf{t}}|\mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) \cdot \widetilde{\Pi}_{im}(a_{im}^{t} = 1, \mathbf{a}_{-\mathbf{im}}^{\mathbf{t}}, \mathbf{s}_{\mathbf{m}}^{\mathbf{t}})$ as:

$$\sum_{\mathbf{a}_{-\mathbf{im}}^{\mathbf{t}}} \sigma_{im}(\mathbf{a}_{-\mathbf{im}}^{\mathbf{t}} | \mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) \cdot \widetilde{\Pi}_{im}(a_{im}^{t} = 1, \mathbf{a}_{-\mathbf{im}}^{\mathbf{t}}, \mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) = K(\mathbf{s}_{\mathbf{m}}^{\mathbf{t}}, a_{im}^{t} = 1)' \Theta_{im} + \beta \cdot \mathbf{Z}_{\mathbf{m}}^{\mathbf{1}}(\mathbf{s}_{\mathbf{m}}^{\mathbf{t}+1} | \mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) \cdot [\Delta \cdot \boldsymbol{\Psi}_{\mathbf{im}} + \Delta \cdot K \cdot \Theta_{im}]$$

This means that the terms inside the inner curly brackets of (5) reduces to:

$$\begin{split} \sum_{\mathbf{a}_{-i\mathbf{m}}^{\mathbf{t}}} \sigma_{im}(\mathbf{a}_{-i\mathbf{m}}^{\mathbf{t}}|\mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) \cdot \left[\widetilde{\Pi}_{im}(a_{im}^{t}=1, \mathbf{a}_{-i\mathbf{m}}^{\mathbf{t}}, \mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) - \widetilde{\Pi}_{im}(a_{im}^{t}=0, \mathbf{a}_{-i\mathbf{m}}^{\mathbf{t}}, \mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) \right] = \\ \begin{cases} \left[K(\mathbf{s}_{\mathbf{m}}^{\mathbf{t}}, a_{im}^{t}=1) - K(\mathbf{s}_{\mathbf{m}}^{\mathbf{t}}, a_{im}^{t}=0)\right]' + \\ \beta \cdot \left[\mathbf{Z}_{\mathbf{m}}^{\mathbf{1}}(\mathbf{s}_{\mathbf{m}}^{\mathbf{t}+1}|\mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) - \mathbf{Z}_{\mathbf{m}}^{\mathbf{0}}(\mathbf{s}_{\mathbf{m}}^{\mathbf{t}+1}|\mathbf{s}_{\mathbf{m}}^{\mathbf{t}})\right] \cdot \Delta \cdot K \end{cases} \\ \beta \cdot \left[\mathbf{Z}_{\mathbf{m}}^{\mathbf{1}}(\mathbf{s}_{\mathbf{m}}^{\mathbf{t}+1}|\mathbf{s}_{\mathbf{m}}^{\mathbf{t}}) - \mathbf{Z}_{\mathbf{m}}^{\mathbf{0}}(\mathbf{s}_{\mathbf{m}}^{\mathbf{t}+1}|\mathbf{s}_{\mathbf{m}}^{\mathbf{t}})\right] \cdot \Delta \cdot \Psi_{i\mathbf{m}} \end{split}$$

Insert this formula into (5) and take logs and then write the LHS of the linearized expression as $y_i(\mathbf{s_m^t}) = ln \left\{ ln \left[\frac{1}{1 - H(a_i^t = 1 | \mathbf{s_t}; \sigma_i(\cdot), \mathbf{\Theta})} \right] \right\}$. Therefore:

$$y_{i}(\mathbf{s_{m}^{t}}) = \beta \cdot \left[\mathbf{Z_{m}^{t}}(\mathbf{s_{m}^{t+1}}|\mathbf{s_{m}^{t}}) - \mathbf{Z_{m}^{0}}(\mathbf{s_{m}^{t+1}}|\mathbf{s_{m}^{t}}) \right] \cdot \Delta \cdot \Psi_{im} + \\ \begin{cases} \left[K(\mathbf{s_{m}^{t}}, a_{im}^{t} = 1) - K(\mathbf{s_{m}^{t}}, a_{im}^{t} = 0) \right]' + \\ \beta \cdot \left[\mathbf{Z_{m}^{1}}(\mathbf{s_{m}^{t+1}}|\mathbf{s_{m}^{t}}) - \mathbf{Z_{m}^{0}}(\mathbf{s_{m}^{t+1}}|\mathbf{s_{m}^{t}}) \right] \cdot \Delta \cdot K \end{cases} \\ \Theta_{im} \end{cases}$$

Given that elements in Ψ_{im} are known we can write $y_i^*(\mathbf{s}_m^t) = y_i(\mathbf{s}_m^t) - \beta \cdot [\mathbf{Z}_m^1(\mathbf{s}_m^{t+1}|\mathbf{s}_m^t) - \mathbf{Z}_m^0(\mathbf{s}_m^{t+1}|\mathbf{s}_m^t)] \cdot \Delta \cdot \Psi_{im}$. Also, to simplify the notation, use:

$$\left\{ \begin{array}{l} \left[K(\mathbf{s_m^t}, a_{im}^t = 1) - K(\mathbf{s_m^t}, a_{im}^t = 0) \right]' + \\ \beta \cdot \left[\mathbf{Z_m^1}(\mathbf{s_m^{t+1}} | \mathbf{s_m^t}) - \mathbf{Z_m^0}(\mathbf{s_m^{t+1}} | \mathbf{s_m^t}) \right] \cdot \Delta \cdot K \end{array} \right\} = M_i(\mathbf{s_m^t})$$

Therefore, $y_i^*(\mathbf{s_m^t}) = M_i(\mathbf{s_m^t}) \cdot \Theta_{im}$.

■ **Proposition 1.** The equivalence between the ALSE-I and the OLS is straightforward given the linearity of the best response function. The asymptotic distribution of the estimator follows directly from Gourieroux and Monfort (2005).

Appendix 2: Reduced Forms

Table 9: Marginal Effects of n_{mt-1}^{pub} and n_{mt-1}^{pri} on the Entry Probabilities of Private Players (Bradesco and Itau) - Subsample $n^{pub} \ge 1$

	(I)	(II)	(III)	(IV)
N ^o Public	0.11467***	0.12303***	0.15040***	0.14810***
	[0.02]	[0.02]	[0.03]	[0.03]
$N^{\underline{o}}$ Private	-0.04343*	-0.02822	-0.07914^{**}	-0.07935^{**}
	[0.02]	[0.03]	[0.03]	[0.03]
Player Dummy	Yes	Yes	Yes	Yes
Time Dummies	Yes	Yes	Yes	Yes
State Dummies	No	Yes	Yes	Yes
Trend*State Dummies	No	No	Yes	Yes
Transfers, Expenditure, Agric. Prod.	No	No	No	Yes
Observations	9,348	9,164	9,164	9,162
Pseudo R2	0.87	0.88	0.92	0.92

Note: (***) Significant at 1%; (**) significant at 5%; (*) significant at 10%. Marginal effects calculated at the sample means. Clustered standard errors at the municipality level in brackets. All the models have lagged activity, number of public and private competitors and municipality payroll. Subsample $n^{pub} \ge 1$ includes all municipalities that had at least one public player in every period.

	(I)	(II)	(III)	(IV)
N ^o Public	0.19269***	0.18806***	0.20575***	0.21391***
	[0.03]	[0.04]	[0.04]	[0.04]
$N^{\underline{o}}$ Private	-0.05677^{*}	-0.06105^{*}	-0.12490^{***}	-0.11986***
	[0.03]	[0.04]	[0.05]	[0.05]
Player Dummy	Yes	Yes	Yes	Yes
Time Dummies	Yes	Yes	Yes	Yes
State Dummies	No	Yes	Yes	Yes
Trend*State Dummies	No	No	Yes	Yes
Transfers, Expenditure, Agric. Prod.	No	No	No	Yes
Observations	7,301	7,117	7,117	7,115
Pseudo R2	0.87	0.88	0.92	0.92

Table 10: Marginal Effects of n_{mt-1}^{pub} and n_{mt-1}^{pri} on the Entry Probabilities of Private Players (Bradesco and Itau) - Subsample $1 \le n^{pub} \le 3$

Note: (***) Significant at 1%; (**) significant at 5%; (*) significant at 10%. Marginal effects calculated at the sample means. Clustered standard errors at the municipality level in brackets. All the models have lagged activity, number of public and private competitors and municipality payroll. Subsample $1 \le n^{pub} \le 3$ includes all municipalities that had at least one and at most three public players in every period.

Appendix 3: CCPs

	0		· (
	(I)	(II)	(I)	(II)
	Sample:1	988-2010	Sample:1	996-2010
Lagged Activity	6.75^{***}	6.73^{***}	6.88***	7.22***
	[0.09]	[0.10]	[0.12]	[0.14]
Market Payroll	0.03^{***}	0.02^{***}	0.04***	0.02^{***}
	[0.00]	[0.00]	[0.00]	[0.00]
Sample Payroll		0.07^{***}		0.17^{***}
		[0.01]		[0.01]
Market 1		-1.15***		-1.17***
		[0.26]		[0.32]
Market 2		-1.09***		-1.45***
		[0.23]		[0.29]
Market 3		-0.64***		-0.88***
		[0.22]		[0.26]
Dummy BB	1.35^{***}	1.68^{***}	1.82***	2.22^{***}
	[0.11]	[0.13]	[0.14]	[0.17]
Constant	-3.70***	-3.56***	-4.55***	-5.58***
	[0.12]	[0.25]	[0.16]	[0.33]
Observations	20,357	20,357	13,680	13,680
Pseudo R2	0.793	0.796	0.815	0.827

Table 11: CCP Logit for Public Players (BB and CEF)

Note: (***) Significant at 1%; (**) significant at 5%; (*) significant at 10%. Clustered standard errors in brackets. Model I does not include sample payroll and market dummies. Model II includes these variables.

	(I)	(II)	(I)	(II)
	Sample:1	.988-2010	Sample:1	996-2010
Lagged Activity	7.44^{***}	7.32***	8.05***	8.17***
	[0.12]	[0.13]	[0.18]	[0.20]
N Public	0.25^{***}	0.25^{***}	0.46***	0.63^{***}
	[0.05]	[0.07]	[0.06]	[0.08]
N Private	-0.42***	-0.40***	-0.57***	-0.35**
	[0.10]	[0.11]	[0.12]	[0.15]
Market Payroll	0.02***	0.01***	0.01***	0.01***
	[0.00]	[0.00]	[0.00]	[0.00]
Sample Payroll		0.08***		0.03**
		[0.01]		[0.01]
Market 1		-0.42		0.98^{**}
		[0.32]		[0.38]
Market 2		-0.12		0.85^{***}
		[0.26]		[0.32]
Market 3		0.13		0.51^{**}
		[0.21]		[0.26]
Dummy Bradesco	0.05	0.04	-0.51***	-0.52***
	[0.11]	[0.11]	[0.13]	[0.13]
Constant	-3.84***	-4.43***	-3.41***	-4.84***
	[0.11]	[0.39]	[0.13]	[0.46]
Observations	$15,\!919$	$15,\!919$	10,595	10,595
Pseudo R2	0.828	0.831	0.830	0.831

Table 12: CCP Logit for Private Players (Bradesco and Itau)

Note: (***) Significant at 1%; (**) significant at 5%; (*) significant at 10%. Clustered standard errors in brackets. Model I does not include sample payroll and market dummies. Model II includes these variables.